normal to

57. $\boxed{xy + 2x - y = 0}$ and $\parallel$ $2x+y=0$

$y=-2x$

$xy'+y+2-y'=0$

$xy'-y'=-y-2$

$y'(x-1)=-y-2$

$y'=\frac{-y-2}{x-1}$

slope of tan

normal slope $=-2$ so slope of tan $=\frac{1}{2}$

$$\boxed{\frac{-y-2}{x-1} = \cancel{-2}\ \frac{1}{2}}$$

$-y-2=\frac{1}{2}(x-1)$

$-y=\frac{1}{2}(x-1)+2$

$y=-\frac{1}{2}(x-1)-2$

$x=-1$ or $x=3$

$y=-1$ $\quad$ $y=-3$

normal lines:

$y=-2(x+1)-1$

$y=-2(x-3)-3$

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56. normal to $\boxed{x^2+2xy-3y^2=0}$ at $(1,1)$ intersects at what other point

$2x+2xy'+2y-6yy'=0$

$2xy'-6yy'=-2x-2y$

$y'(2x-6y)=-2x-2y$

$y'=\frac{-2x-2y}{2x-6y}\Big|_{(1,1)}=\frac{-4}{-4}=1=m_{tan}$

normal: $\boxed{y=-1(x-1)+1}$

Sep 28-9:30 AM

3.8 Derivatives of inverse trig functions

Derivative of the Arcsine

$\boxed{y=\sin^{-1}(x) \text{ means } x=\sin(y)}$

$y=\sin x$

inverse: swap x & y

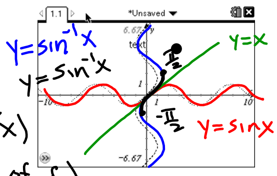
$x=\sin y$

solve for y

$y=\sin^{-1}x$

$y=f^{-1}(x)$

(inverse of f)



restrict the range to make y=arcsine(x) a function

$\sin^{-1}(1)=y$ means $\sin y=1$

$\sin^{-1}(1)=\frac{\pi}{2}$ $\quad$ $y=\frac{\pi}{2}$

$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)=\frac{\pi}{4}$

Oct 4-6:55 PM

$x^2+u^2=1$

$u=\sqrt{1-x^2}$

$y=\sin^{-1}x$ means $x=\sin y$ implicit diff

$y'=?$

$1=\cos y\cdot y'$

$y'=\frac{1}{\cos y}$

$y'=\frac{1}{\sqrt{1-x^2}}$

$$\boxed{\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}}}$$

$$\boxed{\frac{d}{dx}\sin^{-1}(u)=\frac{1}{\sqrt{1-u^2}}\cdot\frac{du}{dx}}$$

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$$\frac{d}{dx}\left(\sin^{-1} x^2\right) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

$$\frac{d}{dx}\left(\sin^{-1} \frac{\sqrt{x}}{3}\right) = \frac{1}{\sqrt{1-\left(\frac{\sqrt{x}}{3}\right)^2}} \cdot \frac{3 \cdot \frac{1}{2} x^{-\frac{1}{2}} - \sqrt{x} \cdot 0}{3^2}$$

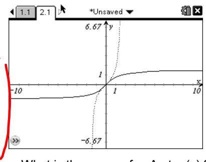
$\frac{1}{3} x^{\frac{1}{2}}$

$\frac{1}{3} \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{6\sqrt{x}}$

$$= \frac{1}{\sqrt{1-\frac{x}{9}}} \cdot \frac{1}{6\sqrt{x}}$$

Derivative of the Arctangent

$$\boxed{\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \cdot \frac{du}{dx}}$$



What is the range of y=Arctan(x) ?

A particle moves along the x-axis so that its positon at any time t>=0 is $x(t) = \tan^{-1}\sqrt{t}$ What is the velocity of the particle when t=16?

$$v = \frac{dx}{dt} = \frac{1}{1+(\sqrt{t})^2} \cdot \frac{1}{2} t^{-\frac{1}{2}} \Big|_{t=16}$$

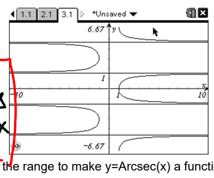
$$= \frac{1}{1+(\sqrt{16})^2} \bullet \frac{1}{2} (16)^{-\frac{1}{2}}$$

$16^{-\frac{1}{2}} = \frac{1}{\sqrt{16}}$

$$= \frac{1}{17} \cdot \frac{1}{8} = \frac{1}{136}$$

Derivative of the Arcsecant

$$\boxed{\frac{d}{dx} \sec^{-1} u = \frac{1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}}$$



restrict the range to make y=Arcsec(x) a function

$$\frac{d}{dx}\sec^{-1}\left(5x^4\right) = \frac{1}{|5x^4|\sqrt{(5x^4)^2-1}} \cdot 20x^3$$

$$= \frac{20x^3}{5x^4\sqrt{25x^8-1}} = \frac{4}{x\sqrt{25x^8-1}}$$

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Derivatives of the other three

$\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$ $\qquad \frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$

$\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$ $\qquad \frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2}$

$\csc^{-1}x = \frac{\pi}{2} - \sec^{-1}x$ $\qquad \frac{d}{dx}\csc^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$

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Derivative of an inverse function

If *f* and *g* are inverse functions then $g'(x) = \frac{1}{f'(g(x))}$

$f(x)$, $g(x) = f^{-1}(x)$

$f(x) = \sin x$

$g(x) = \sin^{-1}x$

$g'(x) = \frac{1}{\cos y}$

$$\boxed{g'(x) = \frac{1}{f'(y)}}$$

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